

$$1. \quad 2x^3 + 7x^2 + 2x - 3 \quad [3, 5]$$

$$\frac{f(b) - f(a)}{b - a}$$

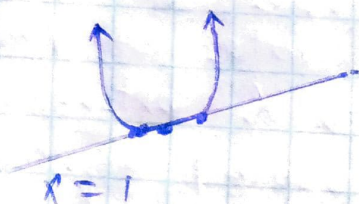
$$f(3) = 2(3)^3 + 7(3)^2 + 2(3) - 3 = 120$$

$$f(5) = 2(5)^3 + 7(5)^2 + 2(5) - 5 = 430$$

$$\frac{f(5) - f(3)}{5 - 3} = \frac{430 - 120}{2} = 155$$

$$2. \quad f(x) = 2x^2 + 3x + 1$$

a) Instantaneous rate of change when $x = 1$



centered interval of 0.01
 $\frac{0.01}{2} = 0.005$

$$1 + 0.005 = 1.005$$

$$1 - 0.005 = 0.995$$

$$[0.995, 1.005]$$

$$\Delta f = \frac{f(1.005) - f(0.995)}{1.005 - 0.995}$$

$$f(1.005) = 2(1.005)^2 + 3(1.005) + 1 = 6.03505$$

$$f(0.995) = 2(0.995)^2 + 3(0.995) + 1 = 5.96505$$

$$1.005 - 0.995$$

$$= \frac{0.07}{0.01} = 7$$

b) Value of the derivative when $x = 1$

$$f(x) = 2x^2 + 3x + 1$$

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$$

$$\begin{aligned} f(1+h) &= 2(1+h)^2 + 3(1+h) + 1 \\ &= 2(1 + 2h + h^2) + 3(1+h) + 1 \\ &= 2 + 4h + 2h^2 + 3 + 3h + 1 \end{aligned}$$

$$f(1) = 2(1)^2 = 2h^2 + 7h + 6 + 3(1) + 1$$

$$f(1) = 6$$

3) A secant line on a curve is a straight line that intersects ~~line~~ the curve at two points. To find the [average] rate of change of a function with respect to x between $A(a, f(a))$ and $B(b, f(b))$, following equation is used $(f(b) - f(a)) / (b - a)$

A tangent line to a curve is a straight line that touches the curve at one point. The point where the line touches the curve is called the tangent point. To calculate the instantaneous rate of change at the tangent point, we must calculate the slope of a tangent line.

$$4(a) \quad dt = -t^2 + 8t + 1$$

The slope is given as $\frac{y_2 - y_1}{x_2 - x_1}$

$$\text{let } x_1 = 1 \quad \text{and} \quad x_2 = 3$$

$$y_2 = (-3)^2 + 8(3) + 1 = 16$$

$$y_1 = (-1)^2 + 8(1) + 1 = 8$$

$$\frac{16 - 8}{3 - 1} = \frac{8}{2} = 4$$

The rate of change is 4

$$b) \quad d(t) = -t^2 + 8t + 1$$

At 2 seconds

$$x = 2$$

$$= 2t - 8$$

$$\frac{dy}{dx} = \frac{f(g+h) - f(h)}{h}$$

$$g = 2$$

$$= \frac{[2(2+h) - 8 - (2(2) - 8)]}{h}$$

$$= \frac{[2(2+h)^2 - 8] - [2(2)^2 - 8]}{h}$$

$$= \frac{2(4 + 2h + h^2) - 8 - [8 - 8]}{h}$$

$$\frac{2h^2 + 4h + 8 - 8}{h}$$

$$\frac{h(2h + 4)}{h}$$

as $h \rightarrow 0$

$$d(2) = 4$$

$$5a) \lim_{x \rightarrow 2} (3x^3 + 7x - 16)$$

$$= 3(2)^3 + 7(2) - 16$$

$$= 22$$

b) What is the meaning of this value.
As $x \rightarrow 2$, the limit of the equation is 22.

$$6a) \lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$$

$$\lim_{x \rightarrow 4} \frac{4^2 - 16}{4 - 4}$$

$$\lim_{x \rightarrow 4} \frac{16 - 16}{4 - 4}$$

$$= \frac{0}{0} = \infty$$

$$b) \lim_{x \rightarrow \infty} \frac{8x^3 - 5x^2 + 17}{6x^3 - 2x^2 + 4x}$$

$$\lim_{x \rightarrow \infty} \frac{x^3 (8 - 5/x + 17/x^3)}{x^3 (6 + 2/x - 4/x^2)}$$

$$\lim_{x \rightarrow \infty} \frac{8 - 5/x + 17/x^3}{6 + 2/x - 4/x^2} = \frac{8}{6} = \frac{4}{3}$$

$$c) \lim_{h \rightarrow 0} \frac{\sqrt{49+h} - 7}{h}$$

$$\lim_{h \rightarrow 0} \frac{49 + h + 7(\sqrt{49+h} - 7) - 49}{h(\sqrt{49+h} + 7)}$$

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{49+h} + 7} =$$

$$\lim_{h \rightarrow 0} \frac{1}{7+7} = \frac{1}{14}$$

$$d. \lim_{y \rightarrow 2} \frac{y^2 - 8}{2y^2 - 7y + 12}$$

$$= \frac{2^3 - 8}{2 \times 2^2 - 7 \times 2 + 12} = \frac{0}{6} = 0$$

$$e.) \lim_{h \rightarrow 0} \frac{\frac{4}{2+h} - 2}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4 - 2(2+h)}{2+h} = \lim_{h \rightarrow 0} \frac{4+4-2(2+h)}{2+h}$$

$$\lim_{h \rightarrow 0} \frac{\frac{2h}{2+h}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{-2h^2}{2+h} = \frac{0}{2+0} = \frac{0}{2} = 0$$

$$7. f(x) = 2x^2 + 3x$$

Point $(-2, 2)$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$f(x) = 2x^2 + 3x$$

$$f(x+h) = 2(x+h)^2 + 3(x+h)$$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{2(x+h)^2 + 3(x+h) - (2x^2 + 3x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{2(x^2 + 2xh + h^2) + 3(x+h) - (2x^2 + 3x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{2x^2 + 4xh + 2h^2 + 3x + 3h - (2x^2 + 3x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{2h^2 + 4xh + 3h}{h} \right]$$

$$\lim_{h \rightarrow 0} \left[\frac{h(2h + 4x + 3)}{h} \right]$$

$$\lim_{h \rightarrow 0} (2h + 4x + 3) = 4x + 3$$

$$4(-2) + 3 = -5$$

$$y - y_1 = m(x - x_1)$$

$$y - (-2) = -5(x - (-2)) \quad y + 2 = -5(x + 2)$$

$$y = -5(x + 2) + 2$$

$$y = -5x - 10 + 2$$

$$y = -5x - 8$$

8. Find the derivative

$$f(x) = x^2 - 10x + 3$$

$$= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = x^2 - 10x + 3$$

$$f(x+h) = (x+h)^2 - 10(x+h) + 3$$

$$= x^2 + 2xh + h^2 - 10x - 10h + 3$$

$$d \quad \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - 10x - 10h + 3 - (x^2 - 10x + 3)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h^2 + 2xh - 10h}{h}$$

$$\lim_{h \rightarrow 0} \frac{h(h + 2x - 10)}{h}$$

$$\lim_{h \rightarrow 0} h + 2x - 10 = 2x - 10$$

$$f'(x) = 2x - 10$$

9. $f(x) = x^2 - 7x + 12$
point (2, 2)

Find the eqn of the tangent line.

$$f(x) = x^2 - 7x + 12$$

$$f(x+h) = (x+h)^2 - 7(x+h) + 12$$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{(x+h)^2 - 7(x+h) + 12 - (x^2 - 7x + 12)}{h} \right]$$

$$\lim_{h \rightarrow 0} \left[\frac{x^2 + 2xh + h^2 - 7x - 7h + 12 - x^2 + 7x - 12}{h} \right]$$

$$\lim_{h \rightarrow 0} \left[\frac{h^2 + 2xh - 7h}{h} \right]$$

$$\lim_{h \rightarrow 0} \left[\frac{h^2 + 2xh - 7h}{h} \right]$$

$$\lim_{h \rightarrow 0} \left[\frac{h(h + 2x - 7)}{h} \right]$$

$$\lim_{h \rightarrow 0} [h + 2x - 7] = 2x - 7 \quad m = 2(2) - 7 = -3$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -3(x - 2)$$

$$y - 2 = -3x + 6$$

$$y = -3x + 8$$

10. $y = 3x^2 + 2x$ Parallel to a line $m = 10$

$$y = 10x - 2$$

slope of $y = 3x^2 + 2x$

$$\frac{dy}{dx} = 6x + 2$$

$$6x + 2 = 10$$

$$\frac{6x}{6} = \frac{8}{6}$$

$$x = \frac{8}{6} = 1.333$$

$$y = 3\left(\frac{8}{6}\right)^2 + 2\left(\frac{8}{6}\right) =$$

$$y = 32\frac{1}{6} + 16\frac{1}{6} = 48\frac{1}{6} = 8$$

$$(1.33, 8)$$